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Dynamical phase diagram of the random field Ising model

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Abstract. The stationary states of the random-field Ising model are determined through the master equation approach, where the contact with the heat bath is simulated by the Glauber stochastic dynamics. The phase diagram of the model is constructed from the stationary values of the magnetization as a function of temperature and field amplitude. The continuous phase transitions coincide with the equilibrium ones, while the first-order transitions occur at fields larger than the corresponding values at equilibrium. The difference between the fields at the limit of stability of the ordered phase and that of the equilibrium is maximum at zero temperature and vanishes at the tricritical point. We also find the mean field time auto-correlation function at the stationary states of the model.

PACS. 64.60.Ht Dynamic critical phenomena

1 Introduction

The random-field Ising model (RFIM) has received much attention in recent years, especially concerning its lower critical dimensionality [1]. Now, it is well known [2–4] that the lower critical dimensionality of this model is two. Many other important questions related to this model can be found in the literature, both theoretical [5,6] and experimentally [7–9]. On the other hand, the time evolution of the thermodynamical systems can be studied by establishing the dynamical stochastic process that simulates the contact with the heat bath. In general, for spin models, we describe the evolution of the probability of the states of the system through the master equation [10], and we choose a transition rate between states, according to the Glauber stochastic process [11]. In this work we consider a ferromagnetic mean-field Hamiltonian, subject to a quenched random field at each site of the lattice. For a distribution of the random fields which is of the bimodal type (sum of two δ functions), Aharony [12] has shown in the mean field approximation that the equilibrium phase diagram of this model exhibits a tricritical point. As we will show below, the dynamical phase diagram we obtain also exhibits the same line of continuous transitions as in the work of Aharony. However, the first-order transition line gives us only the limit of stability of the ordered phase. The stationary solutions of the master equation are not the equilibrium thermodynamic solutions. The difference between the dynamical critical field and the equilibrium one vanishes exactly at the tricritical point.

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2 The model and the calculations

The ferromagnetic mean-field Hamiltonian for the kinetic Ising model in a lattice with N sites is given by

$$H = -\frac{J}{N}\sum_{(i,j)}\sigma_i\sigma_j - \sum_i H_i\sigma_i, \tag{1}$$

where the sum is over all pairs of spins, $\sigma_i = \pm 1$, $J > 0$, and H_i is the quenched local random field at site i. The distribution of the random fields is given by the following bimodal distribution

$$P(H_i) = \frac{1}{2}[\delta(H_i + H_0) + \delta(H_i - H_0)], \tag{2}$$

where H_0 is the magnitude of the random field. The phase diagram of this model was studied by Aharony [12] in the thermodynamical equilibrium. He found that the model exhibits a tricritical point which separates the line of continuous transitions from that of first-order transitions. The coordinates of the tricritical point are given by $\tau = k_{\rm B}T/J = 2/3$ and $h = H/J = 0.43$. Now, we consider the time evolution of the probability states of this system, according to the Glauber stochastic process [11], with the following prescription for the transition rate:

$$W(\sigma_i) = \frac{1}{2}[1 - \sigma_i \tanh(\beta E_i)], \tag{3}$$

where

$$E_i = \frac{J}{N}\sum_{j\neq i}\sigma_j + H_i. \tag{4}$$

From the master equation associated to this stochastic process we can derive the equation for the time evolution

of the mean local magnetization:

$$\frac{\partial\langle\sigma_i\rangle}{\partial t} = -\langle\sigma_i\rangle + \langle\tanh(\beta E_i)\rangle, \tag{5}$$

for a given distribution of the random fields. If $N \to \infty$, we can write that

$$\frac{1}{N}\sum_{i\neq j}\sigma_i = \langle\sigma_i\rangle. \tag{6}$$

Then, taking the mean with respect to the distribution of the random fields,

$$m = \langle\langle\sigma_i\rangle\rangle = \int_{-\infty}^{\infty}\langle\sigma_i\rangle P(H_i)\mathrm{d}H_i, \tag{7}$$

we arrive at the following equation for the time evolution of m:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -m + \frac{1}{2}[\tanh(\beta Jm + \beta H_0) + \tanh(\beta Jm - \beta H_0)]. \tag{8}$$

We can also calculate the time-delayed spin correlation function in this mean field approach. Consider a fixed time t, which will be taken as the initial time, and a given time interval τ. We define the time correlation function for the spins $\sigma_i(t)$ and $\sigma_j(t+\tau)$ as

$$\langle\sigma_i(t)\sigma_j(t+\tau)\rangle = \sum_{\sigma,\sigma'} P(\sigma_1,\sigma_2,\ldots,\sigma_N,t)\sigma_i(t) \times p(\sigma_1,\sigma_2,\ldots,\sigma_N|\sigma'_1,\sigma'_2,\ldots,\sigma'_N)\sigma'_j(t) \tag{9}$$

where $P(\sigma_1,\sigma_2,\ldots,\sigma_N,t)$ is the probability associated with the initial configuration $\{\sigma\}$ at time t and $p(\sigma_1, \sigma_2, \ldots, \sigma_N|\sigma'_1,\sigma'_2,\ldots,\sigma'_N)$ is the conditional probability to find the configuration $\{\sigma\}$ at the later time $(t+\tau)$. If we derive the time correlation function with respect to τ, we obtain, after some algebraic manipulations the following equation

$$\frac{\mathrm{d}\langle\sigma_i(t)\sigma_j(t+\tau)\rangle}{\mathrm{d}\tau} = -2\langle\sigma_i(t)\sigma_k(t+\tau)W(\sigma_i(t))\rangle. \tag{10}$$

Considering equations (3, 4, 6) we can write

$$\frac{\mathrm{d}\langle\langle\sigma_i(t)\sigma_j(t+\tau)\rangle\rangle}{\mathrm{d}\tau} = \langle\langle\sigma_i(t)\sigma_j(t+\tau)\rangle\rangle + \langle\langle\sigma_j(t+\tau)\tanh(\beta J\langle\sigma_i(t)\rangle + \beta H_i)\rangle\rangle, \tag{11}$$

where the mean with respect to the quenched distribution of random-fields is already indicated.

3 Results and discussions

We have numerically solved the equation (8) for each pair of values of temperature and intensity of the random field. We have chosen the initial condition as being $m(0) = 1$. The time to attain the stationary states depends on the selected values of the temperature and of the random field. For the values of the reduced temperature $\tau = \frac{k_\mathrm{B}T}{J} \geq \frac{2}{3}$ the magnetization goes continuously to zero as we change the magnitude of the random field. For instance, we show in Figure 1 the behavior of the stationary values of the magnetization as a function of the reduced intensity of the random field $h = H_0/J$, for the temperature $\tau = 0.75$. For this temperature, the critical value of the field is $h_\mathrm{c} = 0.41$. The critical stationary values we obtain agree with those found by Aharony. The time to achieve the stationary states becomes extremely large as we approach the critical field. This result was already expected because near the transition point the magnetization is very small, and if we linearize equation (8) we obtain

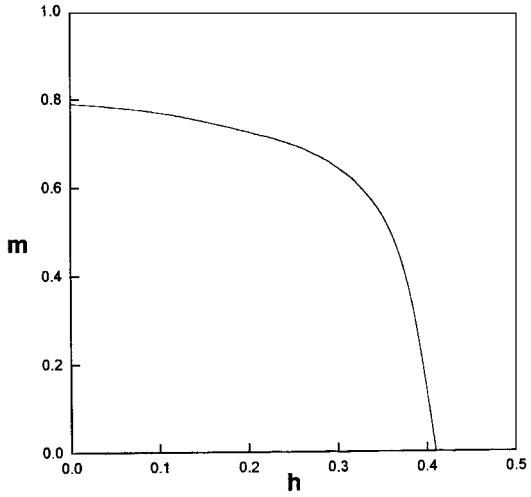

Fig. 1. Stationary magnetization m as a function of the reduced random-field magnitude $h = H_0/J$. Here we take $\tau = k_\mathrm{B}T/J = 0.75$.

$$\frac{\mathrm{d}m}{\mathrm{d}t} = [1 - \sec^2(\beta H_0)\beta J]m, \tag{12}$$

$$m(t) \approx m(0)\mathrm{e}^{-\frac{t}{\Gamma}}, \tag{13}$$

where

$$\Gamma = \frac{\tau}{\tau - \sec^2\left(\frac{h}{\tau}\right)}. \tag{14}$$

Therefore, near the transition point, the relaxation time Γ to attain the stationary state becomes very large because, at the critical point, the condition $\tau_\mathrm{c} = \sec^2(h_\mathrm{c}/\tau_\mathrm{c})$ must be fulfilled [12] by the critical values.

For temperatures $\tau \leq 2/3$, the behavior of the stationary magnetization for very large times can be seen in Figure 2, where $\tau = 0.30$. We observe that, differently from Figure 2, the stationary magnetization is not continuous as a function of the reduced field h. However, the value of the field at the transition is larger than the corresponding one at the equilibrium. The latter field was obtained equating the free energies of the paramagnetic and ferromagnetic phases. The stationary solutions we obtain here give us only the limit of stability of the ordered phase.

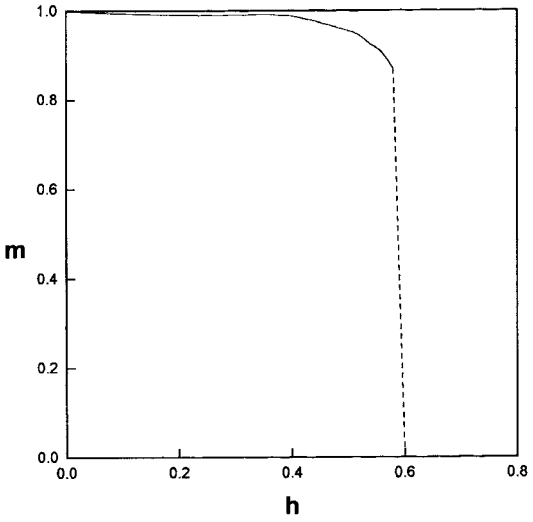

Fig. 2. Stationary magnetization m as a function of the reduced random-field magnitude h. Here we take $\tau = 0.30$.

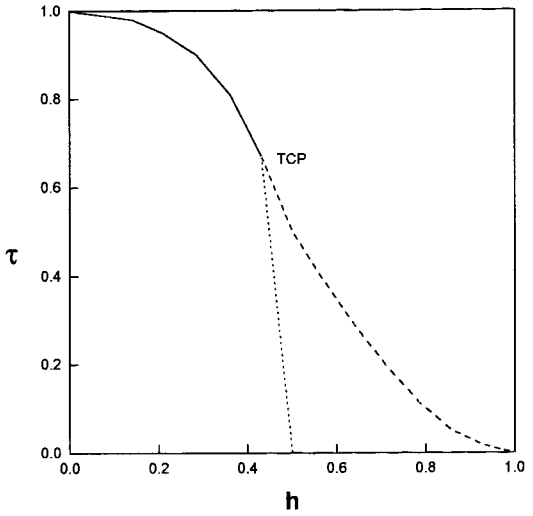

Fig. 3. Phase diagram for the kinetic Ising model in a quenched random-field. The full line corresponds to the continuous phase transitions, the dashed line is associated with the limit of stability of the ferromagnetic phase, and the dotted line represents the equilibrium first-order transitions. TCP is the tricritical point.

We show in Figure 3 the complete phase diagram of the model in the plane τ *versus* h. The line of the continuous phase transitions and the tricritical point (TCP) are the same as in the work of Aharony [12]. However, we have plotted in the same figure the equilibrium first-order phase transitions, given by the dotted line, and the stationary first-order transitions, given by the dashed line. As we can see, the difference between these two transition fields is maximum at zero temperature, and it vanishes exactly at the tricritical point.

The dynamical approach does not give us the free energy of the model, and it is not possible to obtain the stable solutions for this problem. We note that in the region between the dotted and the dashed lines of Figure 3, equation (8) can furnish two stationary solutions: $m = 0$ and $m \neq 0$. As usual for the metastable states, the solution we obtain depends on the initial condition. As in our case we have chosen the initial condition as being $m(0) = 1$, we have got the solution $m \neq 0$ up to the dashed line of Figure 3. Beyond this line, the only stable solution which remains is $m = 0$.

For the evolution of the time correlation function, given by equation (11), let the initial time go to infinity $(t \to \infty)$. In this limit $\langle\langle\sigma_i(t)\rangle\rangle = \langle\langle\sigma_i(t+\tau)\rangle\rangle = m$, where m is the stationary solution of equation (8). Then, taking $i \equiv j$ in equation (11), we obtain the following equation for the auto-correlation function in the limit $(t \to \infty)$:

$$\frac{\mathrm{d}\langle\langle\sigma_i(t)\sigma_i(t+\tau)\rangle\rangle}{\mathrm{d}\tau} = -\langle\langle\sigma_i(t)\sigma_i(t+\tau)\rangle\rangle + m^2. \quad (15)$$

In the paramagnetic phase, where $m = 0$, the solution we obtain is given by

$$\langle\langle\sigma_i(t)\sigma_i(t+\tau)\rangle\rangle = \exp(-\tau), \quad (16)$$

because, at $\tau = 0$, $\langle\langle\sigma_i^2(t)\rangle\rangle = 1$. On the other hand, if the stationary state is ordered, the auto-correlation function is given by

$$\langle\langle\sigma_i(t)\sigma_i(t+\tau)\rangle\rangle = (1 - m^2)\exp(-\tau) + m^2. \quad (17)$$

As to be expected, in the limit of $\tau \to \infty$ the auto-correlation function becomes decoupled, that is,

$$\langle\langle\sigma_i(t)\sigma_i(t+\tau)\rangle\rangle = \langle\langle\sigma_i(t)\rangle\rangle\langle\langle\sigma_i(t+\tau)\rangle\rangle = m^2.$$

Our mean field result for the time auto-correlation function is equivalent to the zeroth-order calculation performed by Sommers [13], which developed a path-integral formalism for the Glauber dynamics and applied it to the Sherrington-Kirkpatrick model of spin glasses.

4 Conclusions

We have studied the dynamical behavior of the Ising model in a quenched random field, with a bimodal distribution for the random fields. The time evolution of the probability states of the system was described by the master equation, with the transition rate given by the stochastic process of the Glauber type. The phase diagram we have obtained in the plane temperature *versus* magnitude of the random field coincides with the equilibrium one only for the continuous transitions between the ferromagnetic and paramagnetic phases. On the other hand, our stationary line for the first-order phase transitions gives transition fields that are greater than the corresponding fields at the thermodynamical equilibrium. The difference between these two transition fields is maximum at zero temperature, and vanishes as we approach the tricritical point. We also have obtained an exponential decay for the time auto-correlation function for the stationary states of the system.

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